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Solutions of the Open Problems in Antimagic Valuations

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Abstract: **A connected graph G is said to be (a, d)-antimagic, for some positive integers a and d, if it's edges admit a labeling by the integers 1,2,…….|E(G)| such that the induced vertex labels consist of an arithmetic progression with the first term a and the common difference d. For n ≥ 8, n ≡ 0 (mod 4), the generalized Petersen graph P(n,2) has a ($\frac{3n}{2}$ + 3, 3)-antimagic labeling. In this paper to prove the following conditions on the generalized Peterson graph.**

(i) If n is even, n ≥ 6 and 2 ≤ k ≤ n/2 -1. Then the generalized Petersen graph P (n,k) is ($\frac{3n}{2}$ +3, 3)- antimagic.

(ii) For n $\equiv 1\ (mod\ 2)$, n ≥ 7 and 1 ≤ k ≤ $\frac{n-1}{2}$, the generalized Petersen graph P (n, k) has a ($\frac{n+7}{2}$, 4) - antimagic labeling.

(iii) For odd n, n ≥ 3, every generalized Petersen graph P (n, 2) has a ($\frac{9n+5}{2}$,2) edge-antimagic total labeling.

(iv) For odd n, n ≥ 5, every generalized Petersen graph P (n, 2) has a super ($\frac{15n+5}{2}$, 1) vertex antimagic total labeling.

Keywords: **Antimagic Valuations, labeling by the integers, generalized Peterson graph.**

1. INTRODUCTION

The graphs considered as finite, undirected and simple. The vertex set of a graph G will be denoted by V(G) (E(G)), respectively. The weight w(v) of a vertex v∈V(G) under an edge labeling f is the sum of values f(e) assigned to all edges incident to a given vertex v. Hartsfield and Ringle[9] introduced the concept of an antimagic graph. An antimagic graph G is a graph whose edges can be labeled with the integers1,2,3,…..,|E(G)| so that the sum of the labels at any given vertex is different from the sum of the labels at any other vertex, that is , no two vertices receive the same weight.Alspach [1] The Classification of Hamiltonian Generalized Petersen Graphs.The concept of an (a,d)-antimagic labelings was introduced by Bodendiek and Walther[8] in 1993.Miller and Baca [4] prove that the generalized Petersen graph P(n,2) is ((3n+6)/2,3) - antimagic for n ≡ 0(mod4). Jirimutu and Wang proved that P (n,2) is ((5n+5) /2, 2) –antimagic for n ≡ 3 (mod 4) and n ≥ 7.Baca, Bertault, MacDougall, Miller, Simanjuntak , and Slamin[162] introduced the notion of (a, d)-Vertex- antimagic total labeling in 2000. In [11] Sugeng and Silaban show : the disjoint union of any number of odd cycles of orders n_1, n_2, …. , n_t, each at least 5, has a super $(3(n_1+n_2+\ldots+n_t) +2, 1)$-vertex-antimagic total labeling; for any odd positive integer t, the disjoint union of t copies of the generalized Petersen graph P (n, 1) has a super (10t +2)n-[n/2]+2, 1)- vertex-antimagic total labeling ; and for any odd positive integers t and n (n≥ 3), the disjoint union of t copies of the generalized Petersen graph P(n, 2) has a super (21tn + 5)/2,1)- vertex-antimagic total labeling.

2. PRELIMINARIES

Let G be a simple graph with order p and size q. A function f :V(G) →{0, 1 …,q } is called a **graceful labeling** if

(i) f is one-to-one.

(ii) The edges receive all the label from 1 to q, where the label of an edge is computed to be the absolute value of the difference between the vertex label at its ends.

Let G be a graph with q edges. G is said to be **magic** if the edges of G can be labeled by the numbers 1, 2, 3, …. ,q so that the sum of the labels of all edges incident with any vertex is the same.

The graphs considered here will be finite, undirected and simple. The vertex set of a graph G will be denoted by V(G) (E(G)), respectively. The weight w(v) of a vertex v∈V(G) under an edge labeling f is the sum of values f(e) assigned to all edges incident to a given vertex v.A connected graph G =(V(G)) is said to be **(a,d)-antimagic** if there exist positive integers a, d and a bijection f : E(G) → $\{1,2,\ldots,|E(G)|\}$ such that the induced mapping

g_f :V(G)→W is also a bijection, where
W = {w(v) : v ∈ V(G)} = { a, a+d, a+2d,…..a+($|V(G)|-1$)d}
is the set of the weights of vertices
if G=(V,E) is (a,d)-antimagic and f:E(G)→{1,2,….,|E(G)|}is a corresponding bijective mapping of G then f is said to be an(a,d)-antimagic labeling of G.

(1) ……….. $\sum_{e\in E(P(n,k))}\rho(e) = \frac{3n(3n+1)}{2}$,

(2) ……… $\sum_{v\in V(P(n,k))} w(v)$ =2na + nd (2n -1).

Clearly, the following equations (1), (2) hold

(3) ……….. 2 $\sum_{e\in E(P(n,k))}\rho(e) = \sum_{v\in V(P(n,k))} w(v)$

(4)………… 3n (3n+1) = 2na + nd (2n – 1).

From the linear Diophantine equation (2) we have d = $\frac{3(3n+1)-2a}{2n-1}$

The minimal value of weight which can be assigned to a vertex of degree three is *a=6* Thus we get the upper bound on the value *d*, i.e., 0 < *d* < $\frac{9}{2}$. This implies that

(5)……… if $n \equiv 0$ (mod 2), then *d* is necessarily odd and the equation (2) has exactly the two different solutions (a, d) =($\frac{7n+4}{2}$,1) or (a ,d) =($\frac{3n}{2}$ +3 ,3) , respectively and

(6)………. if $n \equiv 1$ (mod 2), then *d* is necessarily even and the equation (2) has exactly the two different solutions (a, d) =($\frac{5n+5}{2}$,2) or (a, d) = ($\frac{n+7}{2}$,4),, 2) or , respectively.

Theorem 1. For n ≥ 8, n ≡ 0 (mod 4), the generalized Petersen graph P(n,2) has a ($\frac{3n}{2}$ + 3, 3)-antimagic labeling .[11]

3. MAIN RESULT

In view of Theorem 1 and the result that P(n,k) is ((7n +4) /2, 1)-antimagic if n ≥ 4 is even and k ≤ n/2-1. The conjecture that the generalized Petersen graph P(n, k) is (a,d)- antimagic for all feasible values of a and d. In this paper solved three conjectures and also open problem of [11].

Conjecture 1 If n is even, n ≥ 6 and 2 ≤ k ≤ n/2 -1. Then the generalized Petersen graph P(n,k) is ($\frac{3n}{2}$ +3, 3)-antimagic.

Proof:

Define the edge labeling f of P(n, k), n≡ 0 (mod 4), as follows:

$$f(xy_i) = \begin{cases} \frac{4n+1-i}{2} & if \quad 1 \le n \le n-1 \ is\ odd, i \ne 0. \\ \frac{n}{2}+1 & if \quad i = 2, \\ \frac{5n}{2}-1 & if \quad i = 3, \\ \frac{3n+2-i}{2} & s \ \text{if} \ 4 \le i \le n \ \text{is even}, \end{cases}$$

$$f(x_ix_{i+2}) = \begin{cases} 3n-1 & if\ i = 1, \\ \frac{4n+i+1}{2} & if\ i \equiv 3\ (mod\ 4), i \ge 3, \\ \frac{5n+i+1}{2} & if\ i \equiv 1\ (mod\ 4), i \ge 5, \\ \frac{3n+2i}{2} & if \quad i \equiv o\ (mod\ 2) \end{cases}$$

It is easy to verify that the labeling f uses each integer 1, 2, ….., 3n exactly once and this implies that the labeling f is a bijection from the edge set E(P(n, k) to the set {1, 2, …,3n}. Let us denote the weight (under an labeling f) of vertices x_i and y_i of P(n,2) by

$w(y_i)$ = f (y_iy_{i+1}) + $f(y_{i-1}y_i)$ + $f(x_iy_i)$ for 1≤ i ≤ n,
$w(x_i)$ = $f(x_ix_{i+2})$ + f (x_iy_i) +f $(x_{n+i-2}x_i)$ for 1≤ i ≤ n
with indices taken modulo n.

The weight of vertices of P(n,k) under the edge labeling f constitue the sets

W_1 ={$w(y_i)$: 1≤ i ≤ n } ={$\frac{3n}{2}$ +3i : 1≤ i ≤ n }and

W_2 ={$w(x_i)$:1≤ i ≤ n } = {$\frac{9n}{2}$ +3i :1 ≤ i ≤ n }.

We can see that each vertex of P(n,k) receives exactly one label of weight from $W_1 \cup W_2$ and each number from $W_1 \cup W_2$ is used exactly once as a label of weight and further that the set W =$W_1 \cup W_2$ ={ a, a+d, a+2d, …,a+(2n-1)d }, where a = $\frac{3n}{2}$ +3 and d =3 and finally that the Induced mapping g_f :V(P(n,k))→W is bijective.

Conjecture 2. If n is odd, $n \geq 5$ and $2 \leq k \leq \frac{n-1}{2}$, then the generalized Petersen graph P(n, k) is $(\frac{5n+5}{2}, 2)$- antimagic.

proof: Define the edge labeling f of P(n, k), $n \equiv 1 \pmod 2$, as follows

$$f(x_iy_i) = \begin{cases} \frac{(2+i)}{2} & for \quad i \equiv 0 \pmod 2, \\ \frac{n+2+i}{2} & for \quad i \equiv 1 \pmod 2, \end{cases}$$

$$f(x_iy_{i+1}) = \begin{cases} 2n+1 & for\ i \equiv 0 \pmod 2, \\ \frac{(6n+2+i)}{2} & for \quad i \equiv 0 \pmod 2 \quad i \neq 0, \\ \frac{(5n+2-i)}{2} & for \quad i \equiv 0 \pmod 2. \end{cases}$$

$$f(x_iy_{i+2}) = \begin{cases} n+1 & for\ i = 0, \\ \frac{5n+4-i}{4} & for\ i \equiv 1 \pmod 2, \\ \frac{6n+4-i}{4} & for\ i \equiv 2 \pmod 2, \\ \frac{7n+4-i}{4} & for\ i \equiv 3 \pmod 2, \\ \frac{8n+4-i}{4} & for\ i \equiv 0 \pmod 2. \end{cases}$$

It is easy to verify that the labeling f uses each integer 1, 2, ……. ,3n exactly once and this implies that the labeling f is bijection from the edge set E (P(n, k)) to the set {1, 2, ……,3n}. Let us denote the weights (under an edge labeling f) of vertices x_i and y_i of P(n, k) by $w(y_i) = f(y_iy_{i+1}) + f(y_{i-1}y_i) + f(x_iy_i)$ for $1 \leq i \leq n$,
$w(x_i) = f(x_ix_{i+2}) + f(x_iy_i) + f(x_{n+i-2}x_i)$ for $1 \leq i \leq n$
with indices taken modulo n.
The weight of vertices of P (n, k) under the edge labeling f constitute the sets

$W_1 = \{w(y_i) : 1 \leq i \leq n\} = \{\frac{5n}{2} + 5i : 1 \leq i \leq n\}$ and

$W_2 = \{w(x_i) : 1 \leq i \leq n\} = \{\frac{10n}{2} + 5i : 1 \leq i \leq n\}$.

We can see that each vertex of P(n, k) receives exactly one label of weight from $W_1 \cup W_2$ and each number from $W_1 \cup W_2$ is used exactly once as a label of weigt and further that the set $W = W_1 \cup W_2 = \{a, a+d, a+2d, \dots, a+(2n-1)d\}$, where $a = \frac{5n}{2} + 5$ and d =2 and finally that the induced mapping $g_f : V(P(N, K)) \to W$ is bijective.

Conjecture 3

For $n \equiv 1 \pmod 2$, $n \geq 7$ and $1 \leq k \leq \frac{n-1}{2}$, the generalized Petersen graph P (n, k) has a $(\frac{n+7}{2}, 4)$- antimagic labeling.

Proof:

Define the edge labeling f of P (n, k), $n \equiv 1 \pmod 2$, as follows:

$$f(x_iy_i) = \begin{cases} \frac{(13n-i)}{4} & for \quad i \equiv 1 \pmod 2, \\ \frac{(17n+2+i)}{4} & for \quad i \equiv 1 \pmod 2. \\ \frac{16n+2+i}{4} & for\ i \equiv 3 \pmod 2. \end{cases}$$

$$f(y_iy_{i+1}) = \begin{cases} \frac{(18n+3+i)}{2} & for\ i \equiv 1 \pmod 2, \\ \frac{(17n+3+i)}{2} & for \quad i \equiv 2 \pmod 2, \\ \frac{(20n+3+i)}{2} & for \quad i \equiv 3 \pmod 2. \end{cases}$$

$$f(x_iy_{i+2}) = \begin{cases} \frac{(17n-3-i)}{2} & s\ for\ i \equiv 0 \pmod 2, \\ \frac{(15n-3-i)}{2} & for \quad i \equiv 1 \pmod 2, i \neq n-2, \\ \frac{(17n+1)}{2} & for \quad i = n-2, \end{cases}$$

It is easy to verify that the labeling f uses each interger 1, 2, …., 3n exactly once and this implies that the labeling f is a bijection from the edge set E(P(n, k)) to the set {1, 2, .., 3n }. Let us denote the weight (under an edge labeling f) of vertices x_i and y_i of P(n, k) by
$w(y_i) = f(y_iy_{i+1}) + f(y_{i-1}y_i) + f(x_iy_i)$ for $1 \leq i \leq n$,
$w(x_i) = f(x_ix_{i+2}) + f(x_iy_i) + f(x_{n+i-2}x_i)$ for $1 \leq i \leq n$,
with indices taken modulo n.
The weight of vertices of P (n, k) under the edge labeling f constitute the sets

$W_1 = \{w(y_i) : 1 \leq i \leq n\} = \{\frac{n}{2} + 7i : 1 \leq i \leq n\}$ and

$W_2 = \{w(x_i) : 1 \leq i \leq n\} = \{\frac{4n}{2} + 7i : 1 \leq i \leq n\}$.

We can see that each vertex of P (n, k) receives exactly one label of weight from $W_1 \cup W_2$ and each number from $W_1 \cup W_2$ is used exactly once as a label of weight and further that the set $W = W_1 \cup W_2 = \{a, a+d, a+2d, \dots a+(2n-1)d\}$, where $a = \frac{n+7}{2}$ and d = 4 and finally that the induced mapping $g_f : V(P(n, k)) \to W$ is bijective.

Theorem 2

For odd n, n ≥ 3, every generalized Petersen graph P (n, 2) has a ($\frac{9n+5}{2}$, 2) edge-antimagic total labeling.

Proof:

Label the vertices and edges P (n, 2)

$g(u_i) = f(u_i)$, $g(u_iu_{i+1}) = f(u_iu_{i+1})$

$$g(v_iv_{i+2}) = \begin{cases} \frac{(2n-2-i)}{2} & for\ i \equiv 0\ (mod\ 2), \\ \frac{(n-2-i)}{2} & for\ \ i \equiv 1\ (mod\ 2), i \neq n, \\ n & for\ i = n-2. \end{cases}$$

Case n ≡ 1 (mod 4)

$$g(v_i) = \begin{cases} \frac{(16n-i)}{4} & for\ \ i \equiv 0\ (mod\ 4), \\ \frac{(13n-i)}{4} & for\ \ i \equiv 1\ (mod\ 4), \\ \frac{(14n-i)}{4} & for\ \ i \equiv 2\ (mod\ 4), \\ \frac{(15n-i)}{4} & for\ \ i \equiv 3\ (mod\ 4), \end{cases}$$

$$g(u_iv_i) = \begin{cases} \frac{(18n+2+i)}{4} & for\ \ i \equiv 0\ (mod\ 4), \\ \frac{(17n+2+i)}{4} & for\ \ i \equiv 1\ (mod\ 4), \\ \frac{(16n+2+i)}{4} & for\ i \equiv 2\ (mod\ 4), \\ \frac{19n+2+i}{4} & for\ \ i \equiv 3\ (mod\ 4). \end{cases}$$

Case n $\equiv 3\ (mod\ 4)$

$$g(v_i) = \begin{cases} \frac{(16n-i)}{4} & for\ i \equiv 0\ (mod\ 4), \\ \frac{(15n-i)}{4} & for\ i \equiv 1\ (mod\ 4), \\ \frac{(14n-i)}{4} & for\ \ i \equiv 2\ (mod\ 4), \\ \frac{(13n-i)}{4} & for\ \ i \equiv 3\ (mod\ 4), \end{cases}$$

$$g(u_iv_i) = \begin{cases} \frac{(18n+2+i)}{4} & for\ \ i \equiv 0\ (mod\ 4), \\ \frac{(19n+2+i)}{4} & for\ \ i \equiv 1\ (mod\ 4), \\ \frac{(16n+2+i)}{4} & for\ \ i \equiv 2\ (mod\ 4), \\ \frac{(17n+2+i)}{4} & for\ \ i \equiv 3\ (mod\ 4). \end{cases}$$

The edge- weight of P (n, 2) are

$W_g\ (u_iu_{i+1}) = w_f\ (u_iu_{i+1})$

$$W_g(v_iv_{i+2}) = \begin{cases} \frac{(17n-3)-i}{2} & for\ \ i \equiv 0\ (mod\ 2), \\ \frac{(15n-3)-i}{2} & for\ \ i \equiv 1\ (mod\ 2), \\ \frac{(17n+1)}{2} & for\ \ i = n-2, \end{cases}$$

Case n ≡ 1 (mod 4)

$$w_e(u_iv_i) = \begin{cases} \frac{(19n+3i+1)}{2} & for\ i \equiv 0\ (mod\ 4), \\ \frac{(18n+3+i)}{2} & for\ \ i \equiv 1\ (mod\ 4), \\ \frac{(17n+3+i)}{2} & for\ i \equiv 2\ (mod\ 4), \\ \frac{(20n+3+i)}{2} & for\ \ i \equiv 3\ (mod\ 4) \end{cases}$$

case n≡ 3 (mod 4)

$$w_g(u_iv_i) = \begin{cases} \frac{(19n+3+i)}{2} & for\ \ i \equiv 0\ (mod\ 4), \\ \frac{(20n+3+i)}{2} & for\ \ i \equiv 1\ (mod\ 4), \\ \frac{(17n+3+i)}{2} & for\ \ i \equiv 2\ (mod\ 4), \\ \frac{(18n+3+i)}{2} & for\ \ i \equiv 3\ (mod\ 4). \end{cases}$$

Thus, the set of edge- weight over all edges in P (n, 2) is { a, a +d , ….., (q – 1) d }

Where a = $\frac{1}{2}$ (9n+5) and d =2.

Theorem 3:

For n odd n ≥ 5 every generalized Petersen graph P (n, 2) has a super ($\frac{15n+5}{2}$, 1) vertex antimagic total labeling.

Proof:

Consider the labeling h such that

$$h(u_i) = \begin{cases} \frac{(8n-i)}{2} & for\ i \equiv 0\ (mod\ 2), \\ \frac{(7n\ \ -i)}{2} & for\ \ i \equiv 1\ (mod\ 2), \end{cases}$$

$$h(v_i) = \begin{cases} \frac{(10n-i)}{2} & for\ \ i \equiv 0\ (mod\ 2), \\ \frac{(9n-i)}{2} & for\ \ \ i \equiv 1\ (mod\ 2), \end{cases}$$

$$h(u_iu_{i+1}) = \begin{cases} 2n+1 & for \quad i=0, \\ \frac{(6n+2-i)}{2} & for\ i \equiv 0\ (mod\ 2), i \neq 0, \\ \frac{(5n+2-i)}{2} & for \quad i \equiv 1\ (mod\ 2), \end{cases}$$

$$h(u_iv_i) = \begin{cases} \frac{(2+i)}{2} & for\ i \equiv 0\ (mod\ 2), \\ \frac{(n+2+i)}{2} & for\ i \equiv 1\ (mod\ 2), \end{cases}$$

case $n \equiv 1\ (mod\ 4)$

$$h(v_iv_{i+2}) = \begin{cases} n+1 & for \quad i=0, \\ \frac{(5n+4-i)}{4} & for\ i \equiv 1\ (mod\ 4), \\ \frac{(6n+4-i)}{4} & for\ i \equiv 2\ (mod\ 4), \\ \frac{(7n+4-i)}{4} & for\ i \equiv 3\ (mod\ 4), \\ \frac{(8n+4-i)}{4} & for \quad i \equiv 0\ (mod\ 4), i \neq 0. \end{cases}$$

Case $n \equiv 3\ (mod\ 4)$

$$h(v_iv_{i+2}) = \begin{cases} n+1 & for\ i=0, \\ \frac{(7n+4-i)}{4} & for \quad i \equiv 1\ (mod\ 4), \\ \frac{(6n+4-i)}{4} & for\ i \equiv 2\ (mod\ 4), \\ \frac{(5n+4-i)}{4} & for \quad i \equiv 3\ (mod\ 4), \\ \frac{(8n+4-i)}{4} & for\ i \equiv 0\ (mod\ 4), i \neq 0. \end{cases}$$

Labeling h gives vertex weight w_h

$$w(u_i) = \begin{cases} \frac{1}{2}(17n+3) + (2-i) & for\ i = 0,1. \\ \frac{1}{2}(19n+3) + \frac{1}{2}(4-2i) & for\ i = 2,3.. \end{cases}$$

$$w(v_i) = \begin{cases} \frac{1}{2}(15n+5) + \frac{1}{2}(2-i), & for\ i = 0,2, \\ \frac{1}{2}(16n+4) + \frac{1}{2}(3-i) & for\ i = 1,3,5, \\ \frac{1}{2}(17n+4)\frac{1}{2}(4-i) & for\ i = 4,6,8, \end{cases}$$

Hence, the set of vertex- weights is $\left\{\frac{1}{2}(15n+5), \frac{1}{2}(15n+7) \ldots\ldots\ldots \frac{1}{n}(19n+3)\right\}$

h is super $\left(\frac{1}{2}(15n+5), 1\right)$ is Vertex antimagic total labeling.

4. CONCLUSION

In this paper derived the above conditions on the generalized Petersen graph. The author working in the related field with other various conditions.

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